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# NEW TOOLS IN NONLINEAR SYSTEM ANALYSIS

## (FINAL REPORT)

F49620-00-1-0096, (12.01.1999-03.31.2003)

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### Project Objectives

The project was aimed at developing novel theories for analysis and design of systems exhibiting essentially nonlinear behavior, such as systems utilizing quantized decision making, periodic orbits, switching, etc. The original primary directions of research were: extending the framework of Integral Quadratic Constraints (IQC); studying robustness of periodic trajectories subject to unmodeled dynamical perturbations; efficient analysis of switching piecewise linear systems; model reduction for linear and nonlinear subsystems. A new major research direction has emerged as a result of the work: development of an alternative robust control framework in which finite state stochastic automata serve as nominal system models.

### Summary of Results

IQC system analysis package "iqc $\beta$ " is finished and made available on the Web. New IQC for handling specific nonlinearities are discovered and used to make IQC analysis more accurate.

A new methodology for robustness analysis of forced and self-induced oscillations in dynamically uncertain nonlinear systems is developed and tested.

A new non-conventional methodology of constructive global analysis of piecewise-linear systems (PLS) is developed, based on using *surface Lyapunov functions* instead of ordinary Lyapunov functions. The method is shown to work extremely well in the analysis of benchmark switching systems.

Several new methods for linear system model order reduction (MOR) are discovered, providing error bounds which are superior to those of

Hankel MOR and Balanced Truncation. An procedure for applying these methods in model reduction of nonlinear systems has been proposed.

An alternative robust control design framework is introduced, in which the role of nominal system models is played by finite state automata (FSA). Analogs of the small gain theorem and the full state feedback H-Infinity design algorithm have been derived using standard techniques. Simple hybrid system modeling/analysis/design examples utilizing the technique have been worked out.

### **Personnel Supported**

Alexandre Megretski (PI, Associate Professor), Jorge Goncalves, Reza Oflati-Saber, Chung-Yao Kao (Ph.D. students, theses successfully completed), John Harper (Ph.D. student).

### **Interactions**

Presentations at ACC, CDC, IWOTA, Mohammed Dahleh memorial symposium, Stockholm 2002 symposium on New Directions in Mathematical Systems Theory and Optimization, Cambridge University (UK) control group. Joint research with MIT research groups: Jacob White (VLSI design) and Karen Willcox (computational fluid dynamics).

### **Transitions**

DARPA projects on Software Enabled Control (verification of hybrid systems) and Mixed Circuits Design (model order reduction to enable accurate simulation of electromagnetic interactions).

### **Honors/Awards**

ACC 2000 Best Student Paper Award received by Jorge Goncalves (Ph.D. student at the time).

## Significant Publications

- 1 Goncalves, J., A. Megretski, and M. Dahleh, "Global Stability of Relay Feedback Systems," IEEE Transactions on Automatic Control, Vol. 46(4), 2001
- 2 F. D'Amato, M. Rotea, A. Megretski, and U. Jonsson, "New Results for Analysis of Systems with Repeated Nonlinearities", Automatica, May 2001; 37(5): 739-747
- 3 A. Megretski, "New IQC for Quasi-Concave Nonlinearities", International Journal of Nonlinear and Robust Control, June 2001; 11(7): 603-620
- 4 U.T. Jonsson, C.-Y. Kao, and A. Megretski, "A Cutting-Plane Algorithm for Robustness Analysis of Periodically Time-Varying Systems", IEEE Transactions on Automatic Control, April 2001; 46(4): 579-592
- 5 U.T. Jonsson, C.-Y. Kao, and A. Megretski, "Robustness of Periodic Trajectories", IEEE Transactions on Automatic Control, Nov. 2002; 47(11): 1842-1856
- 6 U.T. Jonsson, C.-Y. Kao, and A. Megretski, "Analysis of Periodically Forced Uncertain Feedback Systems", IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications, vol. 50(2), 2003
- 7 A. Megretski, "Relaxations of Quadratic Programs in Operator Theory and System Analysis", Proceedings of IWOTA 2000, 2001, pp. 365-392.
- 8 A. Megretski, "Optimal Model Order Reduction for Maximal Real Part Norms", Proceedings of the international symposium on New Directions in Mathematical Systems Theory and Optimization.
- 9 A. Megretski, "Robustness of Finite State Automata", Proceedings of Mohammed Dahleh Memorial Symposium, 2001.

## Technical Details

### Integral Quadratic Constraints

In a series of papers, including [2,3], a number of contributions to the method of IQC has been made, enhancing the handling of important nonlinear blocks, such as saturation, relay, and hysteresis. In addition, more flexible version of the general IQC analysis framework is presented, which relaxes the homotopy and boundedness conditions, and is more aligned with the language of the emerging IQC software. The IQC analysis software also made a significant step forward after the package “iqc $\beta$ ” was introduced. “iqc $\beta$ ”, available from

<http://web.mit.edu/ameg/www/home.html>

allows its user to handle easily a variety of problems of robustness analysis.

In the paper [3], a new set of Integral Quadratic Constraints is derived for a class of “rate limiters”, modelled as a series connections of saturation-like memoryless nonlinearities followed by integrators. The result, when used within the standard IQC framework, is expected to be widely useful in nonlinear system analysis. For example, it enables “discrimination” between “saturation-like” and “deadzone-like” nonlinearities and can be used to prove stability of systems with saturation in cases when replacing the saturation block by another memoryless nonlinearity with equivalent slope restrictions makes the whole system unstable. In particular, it is shown that the  $L_2$  gain of a unity feedback system with a rate limiter in the forward loop cannot exceed  $\sqrt{2}$ .

### Robustness of periodic orbits

In [4,5,6], a methodology for robustness analysis of forced and self-induced oscillations in nonlinear systems is being developed. The linearization of the dynamics around a periodic solution to an autonomous ODE has a neutrally stable mode. It is therefore not suitable to directly apply standard techniques for robustness analysis of time-varying systems. It is shown that the system can be transformed in such a way that the neutral mode can be removed from the stability analysis. Any method for robust stability analysis of periodically-varying linear systems can now be applied.

## Surface Lyapunov Functions

Many systems of interest are dynamic systems whose behavior is determined by the interaction of continuous and discrete dynamics. These systems typically contain variables or signals that take values from a continuous set and also variables that take values from a discrete, typically finite set. These continuous or discrete-valued variables or signals depend on independent variables such as time, which may also be continuous or discrete. Such systems are known as Hybrid Systems. Although widely used, not much is known about analysis of hybrid systems. This work attempts to take a step forward in understanding and developing tools to systematically analyze certain classes of hybrid systems. In particular, it focuses on a class of hybrid systems known as Piecewise Linear Systems (PLS). These are characterized by a finite number of affine linear dynamical models together with a set of rules for switching among these models. Even for simple classes of PLS, very little theoretical results are known. More precisely, one typically cannot assess a priori the guaranteed stability, robustness, and performance properties of PLS designs. Rather, any such properties are inferred from extensive computer simulations. In other words, complete and systematic analysis and design methodologies have yet to emerge. In this work, we develop an entirely new constructive global analysis methodology for PLS. This methodology consists in inferring global properties of PLS solely by studying their behavior at switching surfaces associated with PLS. The main idea is to analyze impact maps, i.e., maps from one switching surface to the next switching surface. These maps are proven globally stable by constructing quadratic Lyapunov functions on switching surfaces. Impact maps are known to be "unfriendly" maps in the sense that they are highly nonlinear, multivalued, and not continuous. We found, however, that an impact map induced by an LTI flow between two switching surfaces can be represented as a linear transformation analytically parametrized by a scalar function of the state. Moreover, level sets of this function are convex subsets of linear manifolds. This representation of impact maps allows the search for quadratic Lyapunov functions on switching surfaces to be done by simply solving a set of LMIs. Global asymptotic stability of limit cycles and equilibrium points of PLS can this way be efficiently checked. The classes of PLS analyzed in this work are LTI systems in feedback with an hysteresis, an on/off controller, or a saturation. Although this analysis methodology yields only sufficient criteria of stability, it has shown to be very successful in globally analyzing a large

number of examples with a locally stable limit cycle or equilibrium point. In fact, it is still an open problem whether there exists an example with a globally stable limit cycle or equilibrium point that could not be successfully analyzed with this new methodology. Examples analyzed include systems of relative degree larger than one and of high dimension, for which no other analysis methodology could be applied. We have shown that this methodology can be efficiently applied to not only globally analyze stability of limit cycles and equilibrium points, but also robustness, and performance of PLS. Using similar ideas, performance of on/off systems in the sense that bounded inputs generate bounded outputs, can also be checked. Among those on/off and saturation systems analyzed are systems with unstable nonlinearity sectors for which classical methods like Popov criterion, Zames-Falb criterion, IQCs, fail to analyze. This success in globally analyzing stability, robustness, and performance of certain classes of PLS has shown the power of this new methodology, and suggests its potential towards the analysis of larger and more complex PLS. Results are reported in [1].

### **Alternative Robust Control**

A major objective of the reported research was development of an alternative robust control framework in which finite state stochastic automata serve as basic system models. Within this framework, arbitrary systems are represented as interconnections of "nominal" controlled finite state automata and "uncertain feedback" systems, which can have a continuum of states. Integral constraints (i.e. generalized dissipativity conditions with possibly unknown storage functions) are used to quantify the potential for disturbing effects the uncertain feedbacks can have on the nominal dynamics. Lyapunov functions are used for analysis and control Lyapunov functions are used for design. The general analysis and design algorithms establish practicality of optimal robust design for uncertain finite state automata with tens of thousands of states. Since models of practical interest will have larger numbers of states, model order reduction methods for finite state automata are being developed. Robust stability and robust stabilizability theorems have been proven for uncertain FSA with integral constraints. The complexity of optimal robust design and analysis is found to grow polynomially with the cardinality of the state space. The complexity of optimal robust design does not get reduced when fixed subspaces of control Lyapunov functions are used. The work on finding alternative robust stability certificates, such as Rantzer's

density functions, is under way.

### **Model Order Reduction**

It was recently shown [8] that a polynomial time algorithm exists for the problem of optimal model order reduction in the case when the approximation error measure to be minimized is defined as maximum of the real part of the model mismatch transfer function over a set of sample frequencies. The reduced model obtained this way has provably good quality, on par with that delivered by the Hankel optimal model reduction, which is known to produce very high quality model reduction at a significant computational expences. On the other hand, the computational cost of the new algorithm is a fraction of the cost of Hankel model reduction. The result can be extended to the case of parameter-dependent transfer functions, where the algorithm reduces the model simplification task to a semidefite programm with a small number of decision parameters, employing the technique of sums of squares relaxations by Shor. The parameter-dependent model reduction approach can be applied to certain nonlinear systems by considering some of the state variables as parameters.